Chapter 1

1-5 Spreadsheet Activity

Solving Open Sentences

A spreadsheet is a tool for working with and analyzing numerical data. The data is entered into a table in which each row is numbered and each column is labeled by a letter. You can use a spreadsheet to find solutions of open sentences.

Exercises

Use a spreadsheet to find the solution of each equation using the given replacement set.

1. \(x + 7.5 = 18.3; \{8.8, 9.8, 10.8, 11.8\}\)
2. \(6(x + 2) = 18; \{0, 1, 2, 3, 4, 5\}\)
3. \(4x + 1 = 17; \{0, 1, 2, 3, 4, 5\}\)
4. \(4.9 - x = 2.2; \{2.6, 2.7, 2.8, 2.9, 3.0\}\)
5. \(2.6x = 16.9; \{6.1, 6.3, 6.5, 6.7, 6.9\}\)
6. \(12x - 8 = 22; \{2.1, 2.2, 2.3, 2.4, 2.5, 2.6\}\)

Example

Use a spreadsheet to find the solution for \(4(x - 3) = 32\) if the replacement set is \(\{7, 8, 9, 10, 11, 12\}\).

You can solve the open sentence by replacing \(x\) with each value in the replacement set.

Step 1 Use the first column of the spreadsheet for the replacement set. Enter the numbers using the formula bar. Click on a cell of the spreadsheet, type the number and press ENTER.

Step 2 The second column contains the formula for the left side of the open sentence. To enter a formula, enter an equals sign followed by the formula. Use the name of the cell containing each replacement value to evaluate the formula for that value. For example, in cell B2, the formula contains A2 in place of \(x\).

The solution is the value of \(x\) for which the formula in column B returns 32. The solution is 11.

Exercises

1A. Express the relation \(\{(1, 1), (0, 2), (3, -2)\}\) as a table, a graph, and a mapping.
1B. Determine the domain and the range of the relation. The domain for this relation is \(\{0, 1, 3\}\). The range for this relation is \{\(-2, 1, 2\}\).

Example

a. Express the relation \(\{(1, 1), (0, 2), (3, -2)\}\) as a table, a graph, and a mapping.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
</tr>
</tbody>
</table>

b. Determine the domain and the range of the relation.

The domain for this relation is \(\{0, 1, 3\}\). The range for this relation is \{\(-2, 1, 2\}\).

Exercises

1A. Express the relation \(\{(-2, -1), (3, 3), (4, 3)\}\) as a table, a graph, and a mapping.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

1B. Determine the domain and the range of the relation.

domain \((-2, 3, 4); range \{-1, 3\}\)
Chapter 1

1-6 Study Guide and Intervention (continued)

Relations

Graphs of a Relation  The value of the variable in a relation that is subject to choice is called the independent variable. The variable with a value that is dependent on the value of the independent variable is called the dependent variable. These relations can be graphed without a scale on either axis, and interpreted by analyzing the shape.

Example 1  The graph below represents the height of a football after it is kicked downfield. Identify the independent and the dependent variable for the relation. Then describe what happens in the graph.

Example 2  The graph below represents the price of stock over time. Identify the independent and dependent variable for the relation. Then describe what happens in the graph.

Exercises

Identify the independent and dependent variables for each relation. Then describe what is happening in each graph.

1. The graph represents the speed of a car as it travels to the grocery store.
   Ind: time; dep: speed. The car starts from a standstill, accelerates, then travels at a constant speed for a while. Then it slows down and stops.

2. The graph represents the balance of a savings account over time.
   Ind: time; dep: balance. The account balance has an initial value then it increases as deposits are made. It then stays the same for a while, again increases, and lastly goes to 0 as withdrawals are made.

3. The graph represents the height of a baseball after it is hit.
   Ind: time; dep: height. The ball is hit a certain height above the ground. The height of the ball increases until it reaches its maximum value, then the height decreases until the ball hits the ground.

1-6 Skills Practice

Relations

Express each relation as a table, a graph, and a mapping. Then determine the domain and range.

1. \{(-1, -1), (1, 1), (2, 1), (3, 2)\}
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -1 & -1 \\
   1 & 1 \\
   2 & 1 \\
   3 & 2 \\
   \end{array}
   \]
   D = \{-1, 1, 2, 3\}; R = \{-1, 1, 2\}

2. \{(0, 4), (-4, -4), (-2, 3), (4, 0)\}
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 4 \\
   -4 & -4 \\
   -2 & 3 \\
   4 & 0 \\
   \end{array}
   \]
   D = \{-4, -2, 0, 4\}; R = \{-4, 0, 3, 4\}

3. \{(3, -2), (1, 0), (-2, 4), (3, 1)\}
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   3 & -2 \\
   1 & 0 \\
   -2 & 4 \\
   3 & 1 \\
   \end{array}
   \]
   D = \{-2, 1, 3\}; R = \{-2, 0, 1, 4\}

4. The more hours Maribel works at her job, the larger her paycheck becomes.
   Ind: hours worked, dep: size of paycheck

5. Increasing the price of an item decreases the amount of people willing to buy it.
   Ind: price of an item, dep: number of people willing to buy it
1. Express \{ (1, 3), (2, 1), (3, 2), (4, 3) \} as a table, a graph, and a mapping. Then determine the domain and range.

\[ D = (2, 1, 3, 4); \quad R = \{-2, 1, 3, 4\} \]

2. The graph below represents the height of a tsunami as it travels across an ocean. The longer it travels, the higher the tsunami becomes.

The student repeatedly answers questions and then pauses.

3. The graph below represents a student taking an exam.

4. Express the relation shown in each table, mapping, or graph as a set of ordered pairs.

\[
\begin{align*}
X & \quad Y \\
4 & \quad 3 \\
-1 & \quad 4 \\
3 & \quad -2 \\
-2 & \quad 1
\end{align*}
\]

\[ D = (-2, -1, 3, 4); \quad R = \{-2, 1, 3, 4\} \]

5. Express the relation shown in each table, mapping, or graph as a set of ordered pairs.

\[
\begin{align*}
X & \quad Y \\
0 & \quad 9 \\
-8 & \quad 3 \\
2 & \quad -6 \\
1 & \quad 4
\end{align*}
\]

\[ \{(0, 9), (-8, 3), (2, -6), (1, 4)\} \]

6. Express the relation shown in each table, mapping, or graph as a set of ordered pairs.

\[
\begin{align*}
X & \quad Y \\
4 & \quad 3 \\
3 & \quad 2 \\
3 & \quad 1 \\
-2 & \quad 1
\end{align*}
\]

\[ \{(9, 5), (9, 3), (-6, -5), (4, 3), (8, -5), (8, 7)\} \]

7. BASEBALL The graph shows the number of home runs hit by Andruw Jones of the Atlanta Braves. Express the relation as a set of ordered pairs.

\[
\begin{align*}
X & \quad Y \\
0 & \quad 25 \\
-8 & \quad 36 \\
2 & \quad 29 \\
1 & \quad 34
\end{align*}
\]

\[ \{(0, 25), (03, 36), (04, 29), (05, 51), (06, 41), (07, 26)\}; \quad D = \{02, 03, 04, 05, 06, 07\}; \quad R = \{26, 29, 35, 36, 41, 51\} \]

1. HEALTH The American Heart Association recommends that your target heart rate during exercise should be between 50% and 75% of your maximum heart rate. Use the data in the table below to graph the approximate maximum heart rates for people of given ages.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Maximum Heart Rate (beats per minute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>25</td>
<td>195</td>
</tr>
<tr>
<td>30</td>
<td>190</td>
</tr>
<tr>
<td>35</td>
<td>185</td>
</tr>
<tr>
<td>40</td>
<td>180</td>
</tr>
</tbody>
</table>

2. NATURE Maple syrup is made by collecting sap from sugar maple trees and boiling it down to remove excess water. The graph shows the number of gallons of tree sap required to make different quantities of maple syrup. Express the relation as a set of ordered pairs.

\[
\begin{align*}
X & \quad Y \\
0 & \quad 4 \\
8 & \quad 6 \\
20 & \quad 12 \\
30 & \quad 20 \\
40 & \quad 32
\end{align*}
\]

\[ \{(0, 4), (8, 6), (20, 12), (30, 20), (40, 32)\} \]

3. BAKING Identify the graph that best represents the relationship between the number of cookies and the equivalent number of dozens.

Graph A

Graph B

Graph C

4. DATA COLLECTION Margaret collected data to determine the number of books her schoolmates were bringing home each evening. She recorded her data as a set of ordered pairs. She let \( x \) be the number of textbooks brought home after school, and \( y \) be the number of students with \( x \) textbooks. The relation is shown in the mapping.

a. Express the relation as a set of ordered pairs.

\[ \{(0, 12), (1, 8), (2, 23), (3, 28), (4, 11), (5, 11)\} \]

b. What is the domain of the relation?

\[ \{0, 1, 2, 3, 4, 5\} \]

c. What is the range of the relation?

\[ \{8, 11, 12, 23, 28\} \]
**Even and Odd Functions**

We know that numbers can be either even or odd. It is also true that functions can be defined as even or odd. For a function to be even means that it is symmetric about the y-axis. That is, if you fold the graph along the y-axis, the two halves of the graph match exactly. For a function to be odd means that the function is symmetric about the origin. This means if you rotate the graph using the origin as the center, it will match its original position before completing a full turn.

The function \( y = x^2 \) is an even function. The function \( y = x^5 \) is an odd function. If you rotate the graph 180º the graph will lie on itself.

1. The table below shows the ordered pairs of an even function. Complete the table. Plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-12</th>
<th>-5</th>
<th>-1</th>
<th>1</th>
<th>5</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

2. The table below shows the ordered pairs of an odd function. Complete the table. Plot the points and sketch the graph.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-10</th>
<th>-4</th>
<th>-2</th>
<th>2</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>-2</td>
<td>-4</td>
<td>-8</td>
</tr>
</tbody>
</table>

**Exercises**

Determine whether each relation is a function.

1. Yes
2. Yes
3. No
4. No
5. No
6. Yes
7. Yes
8. Yes
9. Yes
10. Yes
11. No
12. No
Skills Practice

Skills Practice

Functions

Determine whether each relation is a function.

1. yes    2. yes    3. yes    4. yes    5. no    6. no    7. yes    8. yes    9. yes    10. yes

Example

If \( f(x) = 3x - 4 \), find each value.

a. \( f(3) \)

\[
\begin{align*}
\text{Replace } x & \text{ with } 3. \\
= & \frac{9}{4} \\
= & \frac{5}{2}
\end{align*}
\]

b. \( f(-2) \)

\[
\begin{align*}
\text{Replace } x & \text{ with } -2. \\
= & \frac{-6}{4} \\
= & \frac{-3}{2}
\end{align*}
\]

Exercises

If \( f(x) = 2x - 4 \) and \( g(x) = x^2 - 4x \), find each value.

1. \( f(4) \) 2. \( g(-2) \) 3. \( f(-5) \)

4. \( g(-3) \) 5. \( f(0) \) 6. \( g(0) \)

7. \( f(3) - 1 \) 8. \( g\left(\frac{1}{2}\right) \) 9. \( g\left(\frac{3}{2}\right) \)

10. \( f(x^2) \) 11. \( f(k + 1) \) 12. \( g(2n) \)

13. \( f(3x) \) 14. \( f(2) + 3 \) 15. \( g(-4) \)

16. \( 6x - 4 \)
Determine whether each relation is a function.

1. yes

2. no

3. yes

4. yes

5. yes

6. yes

7. yes

8. yes

9. yes

10. yes

11. yes

12. yes

13. yes

14. yes

15. yes

16. yes

If \( f(x) = 2x - 6 \) and \( g(x) = x - 2x^2 \), find each value.

8. \( f(-2) \)

9. \( f(-\frac{1}{2}) \)

10. \( g(-1) \)

11. \( f(-\frac{5}{9}) \)

12. \( f(7) \)

13. \( g(-3) + 13 \)

14. \( f(h + 9) \)

15. \( g(3y) \)

16. \( 2g(b) + 1 \)

\( f(h) = 7.5h \)

\( f(15) = 112.50 \)

\( f(20) = 150 \)

\( f(25) = 187.50 \)

a. Write the equation in function notation. \( f(h) = 7.5h \)

b. Find \( f(15) \), \( f(20) \), and \( f(25) \).

c. Is the relationship a function? Explain. \( f(h) = 7.5h \)

b. If the relation can be represented by the equation \( IR = 12 \), rewrite the equation in function notation so that the resistance \( R \) is a function of the current \( I \).

\( f(I) = \frac{12}{I} \)

c. What is the resistance in a circuit when the current is 0.5 ampere?

24 ohms

4. Travel The cost for cars entering President George Bush Turnpike at Beltline Road is given by the relation \( x = 0.75 \), where \( x \) is the dollar amount for entrance to the toll road and \( y \) is the number of passengers. Determine if this relation is a function. Explain.

This relation is not a function. The graph would be a vertical line, which would not pass the vertical line test.

5. Consumer Choices Aisha just received a $40 paycheck from her new job. She spends some of it buying music online and saves the rest in a bank account. Her savings is given by \( f(x) = 40 - 1.25x \), where \( x \) is the number of songs she downloads at $1.25 per song.

a. Graph the function.

b. Find \( f(3) \), \( f(18) \), and \( f(36) \). What do these values represent?

\( f(3) = 36.25 \); buys 3 songs, saves $36.25

\( f(18) = 17.50 \); buys 18 songs, saves $17.50

\( f(36) = -5 \); sample answer: if she wants to buy 36 songs, she needs $5 extra

c. How many songs can Aisha buy if she wants to save $30? 8 songs
Enrichment

Interpreting Graphs of Functions

Interpret Intercepts and Symmetry

Points where the graph intersects an axis are called intercepts. The $y$-coordinate of the point at which the graph intersects the $y$-axis is called a $y$-intercept. Similarly, the $x$-coordinate of the point at which a graph intersects the $x$-axis is called an $x$-intercept.

A graph possesses line symmetry in a line if each half of the graph on either side of the line matches exactly.

Example

Architecture

The Gateway Arch is a function that approximates the shape of the Gateway Arch, where $x$ is the distance from the center point in feet and $y$ is the height in feet. Identify the function as linear or nonlinear. Then estimate and interpret the intercepts, and describe and interpret any symmetry.

1. Linear or Nonlinear: Since the graph is a curve and not a line, the graph is nonlinear.
2. $y$-Intercept: The intercepts of the graph at about $(0, 630)$, so the $y$-intercept of the graph is about 630. This means that the height of the arch is 630 feet at the center point.
3. $x$-Intercepts: The graph intersects the $x$-axis at about $(-320, 0)$ and $(320, 0)$. So the $x$-intercepts are about $-320$ and $320$. This means that the object touches the ground to the left and right of the center point.
4. Symmetry: The right half of the graph is the mirror image of the left half in the $y$-axis. In the context of the situation, the symmetry of the graph tells you that the arch is symmetric. The height of the arch at any distance to the right of the center is the same as its height that same distance to the left.
5. Identify the function graphed as linear or nonlinear. Then estimate and interpret the intercepts of the graph and any symmetry.

Example 1

Right Whale Population

Linear; the $y$-intercept is 250, so there were 250 right whales in 1987; $x$-intercept is 10, so there will be no right whales after 10 generations; no line symmetry.

Example 2

Stock Price

Nonlinear; $y$-intercept is 0, so there is no change in the stock value at the opening bell; $x$-intercepts are 0 and about 5.5, so there is no change in the value after 0 hours and about 5.5 hours after opening; no line symmetry.

Example 3

Average Gasoline Price

Nonlinear; $y$-intercept about 1, so the average price of gas was about $1 per gallon in 1987; no $x$-intercepts, so there is no time when gas was free; no line symmetry.
Chapter 1

1-8 Study Guide and Intervention (continued)

Interpreting Graphs of Functions

Interpreting Extrema and End Behavior

Identifying the function graphed as linear or nonlinear. Then estimate and interpret the extrema of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.

Example

**Health** The outbreak of the H1N1 virus can be modeled by the graph at the right. Estimate and interpret where the function is positive, negative, increasing, and decreasing, the x-intercepts of any relative extrema, and the end behavior of the graph.

**Positive:** for x between 0 and 42

**Negative:** no parts of domain

This means that the number of reported cases was always positive. This is reasonable because a negative number of cases cannot exist in the context of the situation.

**Increasing:** for x between 0 and 42

**Decreasing:** no parts of domain

The number of reported cases increased each day from the first day of the outbreak.

**Relative Maximum:** at about x = 42

**Relative Minimum:** at x = 0

The extrema of the graph indicate the number of reported cases peaked at about day 42.

**End Behavior:** As x increases, y appears to approach 11,000. As x decreases, y decreases. The end behavior of the graph indicates a maximum number of reported cases of 11,000.

Estimate and interpret where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.

1. **Right Whale Population**

   The population is above 0 for the first 10 generations, and then below 0. A negative population is not reasonable. The population is going down for the entire time. There are no extrema. As the time increases, the population will continue to drop.

2. **Stock Price**

   The stock went down in value for the first 3.2 hours, and then rose until the end of the day. The stock value decreases in value for the first 3.2 hours, and then goes up in value for the remainder of the day. The stock had a relative low value after 3.2 hours and then a relative high value at the end of the day. As the day goes on, the stock increases in value.

3. **Average Gasoline Price**

   The average gasoline price is always positive. It increases for the first few years, decreases until about the 11th year, then increases. The relative minima are at 1 and about 11. The average price appears to increase as time passes.

4. **Height of Golf Ball**

   nonlinear; y-intercept = 0; x-intercepts = 0 and 120; line symmetry x = 60; height was always positive and increased until it was 60 yards from the tee and decreased 60 to 120 yards from the tee; see students’ work for interpretations.

5. **Height of Solar Reflector**

   nonlinear; y-intercept = -6.25; x-intercepts = -12.5 and 12.5; line symmetry about the y-axis; positive for x < 12.5 and x > 12.5; the minimum is -6.25 at 0; see students’ work for interpretations.
1-8 Practice
Interpreting Graphs of Functions

Identify the function graphed as linear or nonlinear. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.

1. HEALTH The graph shows the Calories burned by a 130-pound person swimming freestyle laps as a function of time. Identify the function as linear or nonlinear. Then estimate and interpret the intercepts and the end behavior of the graph.

   Calories Burned
   Time (seconds) [0 2 4 6 8 10]
   Calories (kC) [0 1000 2000 3000]

   Linear; the x- and y-intercepts are 0. This means that no Calories are burned when no time is spent swimming.

   Nonlinear; x-intercept is about 4, so water level was about 43 cm when time started; no line symmetry; water level was always positive and decreased the entire time; graph appears to level off or begin to increase as x increases.

2. TECHNOLOGY The graph below shows the results of a poll that asks Americans whether they used the Internet yesterday. Estimate and interpret where the function is positive, negative, increasing, and decreasing, the x-coordinates of any relative extrema, and the end behavior of the graph.

   Did you use the Internet yesterday?
   Time (h) [0 1 2 4 6 8]
   Yes Responses [0 2 4 8 12 24 36 48 60]
   No Responses [0 90 88 86 82 78 74 70 66 62 58 54 50 46 42 38 34 30 26 22 18 14 10 6 2 0]

   Nonlinear; y-intercept is 43, so, so diver started at 10 m; x-intercept of about 1.8, so diver entered the water after about 1.8 sec.; no line symmetry; height was positive for x < 1.8 and negative for x > 1.8, so diver was above the water until 1.8 sec.; the height increased until max. of 10.5 at 0.3 sec., then it decreased; diver would continue to go down for some time, then would come up.

3. GEOMETRY The graph shows the area y in square centimeters of a rectangle with perimeter 20 centimeters and width x centimeters. Describe and interpret any symmetry in the graph.

   Area (cm^2)
   Width (cm) [0 2 4 6 8 10]
   Height of Diver
   Height Above Water (m) [0 1 2 3 4 5 6 7 8]

   Nonlinear; y-intercept is 10, so diver started at 10 m; x-intercept of about 1.8, so diver entered the water after about 1.8 sec.; no line symmetry; height was positive for x < 1.8 and negative for x > 1.8, so diver was above the water until 1.8 sec.; the height increased until max. of 10.5 at 0.3 sec., then it decreased; diver would continue to go down for some time, then would come up.

4. EDUCATION Identify the function graphed as linear or nonlinear. Then estimate and interpret the intercepts of the graph, any symmetry, where the function is positive, negative, increasing, and decreasing, the x-coordinate of any relative extrema, and the end behavior of the graph.

   U.S. Education Spending
   Years Since January 2005

   Linear; the x- and y-intercepts are 0. This means that no Calories are burned when no time is spent swimming.

   Nonlinear; y-intercept is about 10, so spending was about $10 billion in 1949; no x-intercept; function is positive for all values of x, so education spending has never been $0; function is increasing for all values of x, with no relative maxima or minima; as x-increases, y-increases, so the upward trend in spending is expected to continue.
**1-8 Enrichment**

**Symmetry in Graphs of Functions**

You have seen that the graphs of some functions have line symmetry. Functions that have line symmetry in the y-axis are called **even functions**. The graph of a function can also have point symmetry. Recall that a figure has point symmetry if it can be rotated less than 360° about the point so that the image matches the original figure. Functions that are symmetric about the origin are called **odd functions**.

<table>
<thead>
<tr>
<th>Even Functions</th>
<th>Odd Functions</th>
<th>Neither Even nor Odd</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

The graph of a function cannot be symmetric about the x-axis because the graph would fail the Vertical Line Test.

**Exercises**

Identify the function graphed as **even**, **odd**, or **neither**.

1. even
2. odd
3. even
4. neither
5. even
6. odd
7. neither
8. even